Implications of cognitive studies for teaching physics

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I. INTRODUCTION

Many of us who have taught introductory physics for many years recall with dismay a number of salient experiences: a graph but cannot say what it means; a top student who can solve all the problems but not give an overview or simple derivation; many students of varying abilities who memorize without understanding despite our most carefully crafted and elegant lectures.

As physics teachers who care about physics, we have a tendency to concentrate on the physics content we are teaching. We often are most concerned for those students who are like we were—that small fraction of our students who find physics interesting and exciting and who will be the next generation of professional physicists. But the changes in our society and in the role of technology for the general public mean that we must change the way we are teaching. It no longer suffices to reproduce ourselves. Society has a great need not only for a few technically trained people, but for a large group of individuals who understand science.

During the past decade, data have built up that demonstrate that as physics teachers we fail to make an impact on the way a majority of our students think about the world.1,2 We have redrafted our testing so that the students can succeed and we have then either fooled ourselves that we are teaching them successfully or lowered our standards by eliminating understanding from our definition of successful learning. Alan van Heuvelen3 has remarked that in his study of a typical introductory lecture class, 20% of the students entered the first semester of an introductory calculus-based physics class as Newtonian thinkers. The impact of the course was to increase that number to 25%. If we want to reach all students, we must put a much more attention to how students learn and how they respond to our teaching. We must treat the teaching of physics as a scientific problem.

A few physicists have begun to perform detailed experiments to determine what our students are thinking and what works in the teaching of physics. Some of their articles are of tremendous importance and I believe should be read by every physics teacher (see Refs. 4 and 5, and references therein). But even among these few articles, only a small fraction of the authors attempt to relate their results in a general theoretical framework—to give us a way of thinking about and organizing the results.3 Those of us in physics know well that advancements in science is a continual dance between the partners of theory and experiment, first one leading, then the other. It is not sufficient to collect data into a "wizard's book" of everything that happens. That's not science. Neither is it science to spout high-blown theories untainted by "reality checks." Science must build a coherent and clear picture of what is happening at the same time as it continually confirms and refutes that picture against the real world.

The time has come for us to begin the development of a framework for understanding and talking about student learning. Some of the results of the past few decades in cognitive studies4 begin to provide such a framework.

Cognitive studies focuses on how people understand and learn. It is still an amorphous field, and it is not yet really a single discipline. It overlaps many areas from anthropology to neuropsychology. It may not yet be "a science" as we in physics use the term, but developments in the past few decades have changed drastically what we know about how the mind works.

The issue of how to teach physics is a difficult one: the attempt of a naive student to build a good understanding of physics involves many intricate processes over a long period of time. These processes trend to be much more complex than those most cognitive scholars have addressed. Nonetheless, some of the basic ideas of cognitive studies appear to be both firmly grounded and useful to the teacher of physics.

In this essay I briefly review some of the lessons I have learned from cognitive studies. This is not a review article, but a narrow selection from a small slice of a large field. For those interested in a more substantial introduction to cognitive studies I recommend Howard Gardner's historical overview,5 the collection of articles assembled by Gentner and Stevens,6 and some of the articles in the spirit volume collected by Collins and Smith.12 These will give an entry point into the modern cognitive literature. The book by Inhelder and Piaget13 has lots of discussion of experiments on how adolescents learn physics. Some articles by leading educational specialists also can help link to the existing literature.14 Just for fun, I have to add Donald Norman's delightful book on how people interact with objects in their world.15 For an introduction to the physics education research literature, Arnold Arons's book7 and a few review articles provide an appropriate entry point.

I have grouped what I have learned from the cognitivists into four broad principles with elaborative corollaries. One of the things students of cognitive processes have learned about thinking is that it is fuzzy. The sharp, crisp operations of formal logic or the digital computer are not appropriate models for the way most people think. Therefore, it is not correct to call these principles "theorems" or "laws of cognitive science." Nor is it correct to use them as such. They cannot provide us with hard and fast rules for what to do. Using them incursively without reference to experimental data can lead us to the wrong conclusions. But I have found that they help me to organize my thinking about my students and to refocus my attention. Instead of concentrating only on the organization of the physics content, I now also pay attention to what my students are doing when they interact with a physics course. This is not to suggest that an emphasis on content is somehow unimportant or should be neglected. What we are teaching is important, but it must be viewed in the context of what our students learn.
The fundamental change that has led to the breakthroughs in cognitive studies is the past few decades has been a new willingness to model what is happening in the mind in terms of inferred structures. For the first half of this century, studies of thinking were severely constrained by the "behaviorist" philosophy that one should formulate all one's theories only in terms of direct observable objects. This is like the S-matrix theory of elementary particles which insisted that we should only formulate our theories in terms of observable particles and their scattering amplitudes. Elementary particle physicists only made their breakthrough when they were willing to formulate their ideas in terms of quarks and gluons—particles which could only be inferred and not be seen directly. Cognitive scientists started to make real progress when they began to be willing to formulate how people were thinking in terms of mental processes or models that could not be directly observed or measured.

Principle 1: (Weak form) People tend to form mental patterns. This is the fundamental hypothesis about how the mind works. On some levels, there is direct observation of this mental processing by patternning. For example, it has been demonstrated in detail that we process visual information or a variety of levels to form patterns beginning with the first layer of nerve cells attached to the retina, and the process continues through many stages deep into the brain. I once attended a physics colloquium given by Jerome Lemkin on the subject of blind spots in the visual field. He passed out the standard blind-spot demonstration pages that let us clearly find the blind spot in our eye. We then moved the end of a pencil into our blind spot and saw the end of the pencil disappear as it hit off. Yet there was no "blind spot" in the visual field. The brain fills in the background—here the simple white of a blank page. But if we fill in even a quite complex pattern, if the page had been covered with print, my brain would still have filled in my blind spot with the appropriate pattern. But note that the patternning was not sufficiently sophisticated to produce the "right" answer. My automatic filling in my seeing paper in the blind spot, not the rest of the pencil.

The tendency of the human mind to form patterns is not just limited to the analysis of sensory data. This leads me to state the principle in a stronger (and more relevant) form.

Principle 1: (Strong form) People tend to organize their experiences and observations into patterns or mental models.

I use the term mental model for the collection of mental patterns people build to organize their experiences related to a particular topic. I use the term schema (pl. schemata or schemas) to describe the basic elements of these mental models. I think of a schema as a "chunk" or "object" (in the sense of object-oriented programming). It is a collection of closely linked data together with some processes and rules for use. He is careful in the use of the word "model." It tends to convey something clockwork—a mechanism that has links and rules operating in a well-defined way. These are not the characteristics of our mental models.

The characteristics of mental models and schemas are still vigorously debated. Despite attempts to build a general representational system for mental models, none has yet been widely accepted. However, some results are clear.

Properties of mental models:

- Mental models have the following properties:
  1. They consist of propositions, images, rules of procedure, and statements as to when and how they are to be used.
  2. They may contain contradictory elements.
  3. They may not make sense in isolation.
  4. People may not know how to "run" the procedures present in their mental models.
  5. Mental models are often not highly bound.
  6. Mental models tend to minimize expenditure of mental energy. People tend to do the mental activities—sometimes very time consuming and difficult—in order to avoid a little bit of serious thinking.

This inferred structuring of mental models is distinctly different from what we usually assume when teaching physics. We usually assume that our students either know something or they do not. The view of mental models we learn from cognitive scholars suggests otherwise. It suggests that students may hold contradictory elements in their minds without being aware that they contradict.

I had an interesting experience that illustrated for me vividly the surprising fact that one's mental model may contain contradictory elements. Ron Thornton visited the University of Maryland a few years ago to give a seminar on his now famous work on using the Sonic Ranger to teach the concept of velocity. The Ranger detects the position of an object using sound and can display the position or velocity of the detected object on a computer screen in live time. Thornton set up the Ranger to display velocity and had the computer show a preset pattern (a square wave). He then called me up to the front of the room to steve as a guinea pig and try to walk so my velocity matched the preset pattern.

I had been teaching physics for nearly 20 years and felt perfectly comfortable with the concept of velocity. I did my first trial without thinking; I walked backward until my velocity reached the height of the preset square wave. Then I stopped and my velocity dropped to zero immediately. I asked for another chance, and this time, putting my mental model aside, I was able to reduce the curve without difficulty.

What this experience said to me was that, for normal walking, I still maintained a naive (but appropriate) position-dominated proposition is my mental model of motion. I also had a correct proposition for the concept of velocity, but I had to consciously apply a rule telling me to use it.

I have also had personal experiences illustrating characteristics. I once spent an hour searching through my hard drive and pile of floppy disks to find a short paragraph (three sentences) that I needed again. When I found it, I realized it would have taken me only five minutes to have rewritten it from scratch. A nice example of this characteristic is Donald Norman's study of how people use complex calculators (Ref. 20).

An important aspect of Principle 1 is that people organize their experiences into mental models' with the emphasis on the fact that people must build their own mental models. This is the cornerstone of the educational philosophy known as constructivism. For this reason, I refer to Principle 1 as "The Construction Principle.

An extreme statement of constructivism is:
You cannot teach anybody anything. All you can do as a teacher is to make it easier for your students to learn.

Of course, facilitation can be critical to the learning process. If you want to teach someone a disparaging teaching, but as demanding that we get feedback and evaluations from our students to see what works and what does not. It is good to focus less on what we are teaching, and more on how our students are learning.

Implications

A number of interesting corollaries, elaborations, and implications are relevant for understanding physics teaching come from Principle 1. The first is the realization that what we want our students to get is not simply "the content," but to build their understanding of that content into an accurate and effective mental model.

Corollary 1.1: The goal of physics teaching is to have students build accurate mental models for doing physics. This helps us identify the point of teaching physics. I really want my students to do three things:

(a) develop the ability to reason qualitatively about physical processes;
(b) structure that content into coherent and appropriately organized—and appropriately accessible—mental models;
(c) learn how to apply that model to "do" physics in an expert and creative way.

These goals suggest that we should broaden our evaluation procedures. We traditionally test only the content part of the student's skill in doing physics, usually at least at the introductory level, it should not be surprising that students are not doing the problem solving that is required for an accurate and effective mental model.

Some of the characteristics of mental models clarify what is happening when students make mistakes. Often in listening to my students explain what they think I used to become confused and sometimes frustrated. How can they say x when they know the contradictory principle y? How come they cannot get started on a problem when they certainly know the relevant principle? They just told me to do me two minutes ago! How come they brought up that particular principle now? It is not applicable here! The well-documented characteristics of mental models indicate above help me understand that these sorts of errors are natural and to be expected.

Corollary 1.2: It is not sufficient for students to "know" the relevant correct statements of physics. They also have to be able to gain access to them at the appropriate times; and they have to have methods of cross checking and evaluating to be certain that the result they have called up is truly relevant.

We must also test the underlying mental models that the students are developing. Traditional testing fails to do this, because many systems can produce the correct solution to a problem. Even if the student goes through the same steps as we do, there is no guarantee that their scheme for choosing the steps is the same as ours. I once asked a student who had thought through this problem the same correctly to explain his solution. He replied: "We have used all of the other formulas at the end of the chapter except this one, and the unknown starts with the same letters as is in that formula, so that must be the one to use."

The way we have fooled ourselves with our current testing is that we are interested in what students know. If they do not access the right "information" in an exam, we give them clues and hints in the wording to trigger access.

But since an essential component of a mental model is the process for access to information, we are not testing the complete mental model. The student "has" the information, but it is not in and cannot be used or recalled except in very narrow circumstances.

To find out what our students really know we have to give them the opportunity to explain what they are thinking in words. We must also give them credit for reasoning, and not give partial credit when a student tries to hit the target with a blast of shotgun pellets and accidentally hits a perfectly accurate, and relevant equation among a mass of irrelavances.

To know whether our students access the information in appropriate circumstances we have to give them more realistic problems—ones that relate directly to their real world experiences and do not provide too many "physics clues" that specify an access path for them.

My next corollary relates to the problem that some students often seem to listen to us but not hear what we think we are trying to say.

Corollary 1.3: The student is not a tabula rasa (blank slate). Each one comes to us having had experiences with the physical world and having organized these experiences into mental models.

As physics teachers, we must realize that students come to us with naive mental models of how the physical world works. These are often referred to in the literature on physics education as misconceptions (or misperceptions). Even experienced teachers can be surprised by this. A few years ago, after reading the ground-breaking articles of Halloun and Hestenes' in this journal on students' misconceptions in mechanics, I excitedly related a brief description of the results to one of my colleagues—one whom I know to be a concerned teacher—and a superb lecturer. He was skeptical at the idea that students had trouble learning Newton's first law because they had pre-existing mental models that were friction dominated. He insisted: "Just don't tell them about friction. They won't know about it if you don't tell them." This natural expectation of this experienced teacher is now strongly contradicted by an impressive body of data.

The presence of "false" misconceptions is really not so surprising if we think about our students' previous experience. Why should we be surprised that students think that any moving object will eventually come to a stop? In their direct personal experience that is always the case. It is even the case in the demonstrations we show in class to demonstrate the opposite! When I slide a dry ice levitated puck across the lecture table, I watch it and stop at the end of the table. If I did not, it would slide off the table, bounce, roll a short distance, and stop. Every student knows that. Yet I ask them to focus on a small piece of the demonstration—the stretch of about four or five seconds when the puck is sliding along the table smoothly—and extend that observation to their minds to infinity. The student and the teacher are focusing on different aspects of the same physical phenomena.

Many teachers show surprise in response to the educational physics studies that have gathered the pages of this journal demonstrating that students regularly generalize their mistakes about the nature of the universe. Why should we be surprised that students think cutting off part of a lens will result in their seeing only part of an image? Try it with a magnifying glass! (Yes, I know that is not a real image.)

Where do students get the idea that electricity is some- thing that flows out of the wall and is used up in the object? Why don't they think circuits? Although we do not always


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think about is, most of our students have had extensive experience with electricity by the time they arrive in our classes. When I said the current had to come in one wire and go out the other, one of my students complained: "If all the electricity goes back into the wall, what are we paying for?"

Corollary 1.4: Econo-models must be built. People learn better by doing than by watching something being done.

This is sometimes expressed in the phrase: active leaning works better than passive learning. In most cases, this means that reading textbooks and listening to lectures is a poor way of learning. This should not be taken at university true! As physics teachers, most of us have had the experience of having a few "good" students in our lectures—students for whom listening to a lecture is an active process—a dialogue between themselves and the teacher. Indeed, many of us have been that good student and remember lectures at least some of them-as significant parts of our learning experience. A similar statement can be made about texts. I remember with pleasure working through texts and lecture notes, reorganizing the material, filling in steps, and posing questions for myself to answer. Yet few of my students seem to know how to do this or even to know that this is what I expect them to do. This leads us to think about a fifth corollary.

Corollary 1.5: Many of our students do not have appropriate mental models for what is meant to learn physics. This is a "meta" issue. People build mental models not only for content, but also for how to learn and what actions are appropriate under what circumstances. Most of our students do not know what you and I mean by "doing" science or what we expect them to do. Unfortunately, the most common mental model for learning science in our classes seems to be:

(a) Write down every equation of law the teacher puts on the board that is also in the book.
(b) Memorize that equation along with the list of formualts at the end of each chapter.
(c) Do enough homework and end-of-the-chapter problems to recognize which formula is to be applied to which problem.
(d) Pass the exam by selecting the correct formulas for the problems on the exam.
(e) Erase all information from your brain after the exam to make room for the next set of material.

I used to be flabbergasted to discover that when I stopped a lecture and said, "OK, here is a really important general principle for how to do things. It is not in the book but it comes from my years of experience as a physicist," my students would not write it down or consider it important, even if I wrote it on the board (Well, after all, it was not going to be on the exam.)

I call the bulleted list above "the dead leaves model." It is as if physics were a collection of equations on fallen leaves. One might hold A1=1/(2q^2), another F=ma, and a third F=-ka. There are all considered as of equivalent weight, importance, and structure. The only thing one needs is to do when solving a problem is to flip through one's collection of leaves until one finds the appropriate equation. I would much prefer to have my students see physics as a living tree. I like the term mental ecology to describe the mental model that tells students what mental model to apply in what set of circumstances. It is a more important goal to reshape our students' mental ecologies so that they expand their idea of learning to make it more constructive, take it out of the classroom into their everyday lives, and understand what science is and how to apply it, than is to teach them to parrot back equations or solutions to turn-the crank problems.

One final observation on the first principle is the following:

Construting our own leatures and teaching materials can prove very steep. Each student is a learner who, and things they have no knowledge about at all. In the last area my daughter would say they are "cheater." I like to look at this as an archery target. What they know is the bull's-eye—a compact black area; what they know a little about is a gray area surrounding the black; and the clueless registers a white "rest of the world." To teach them something, I do bee to hit in the gray. A class full of students is a challenge because all of their gray areas are not the same. I want to hit as many of the gray as possible with each paint-tipped shaft of information to turn gray areas black.

In communication studies, an important implication of this corollary is called the "given-new principle." It states that new information should always be presented in a context that is familiar to the reader and the context should be established first. The analogous statement is very important in physics teaching, especially at the introductory level. As physicists with years of training and experience we have a great deal of "context" that our students do not possess. Often we are as fish in water, unaware of this context and that it is missing in our student. There are a number of specifics that we can cite that are given-new problems. We often use terms that students are not familiar with—or use in a different sense than we do. As a part of their study in the way speakers of English build their meaning of the term "force," Lakoff and Johnson classified the characteristics of common metaphors using the term. Among their list of 11 characteristics, eight involved...
the will or intent of an individual! But most of us are so familiar with the technical meaning of force that we are surprised to learn that a significant fraction of our introductory students do not believe that a table exerts a force on a book it is supporting. Why doesn't the book fall through? The table is just "in the way." The problem caused by the interpretation of common speech words for technical ones is not a simple one. I know that the terms "heat," and "temperature" are not really distinguished in common speech and are used interchangeably for the technical terms temperature (average energy per degree of freedom), "internal energy," and heat (flow of internal energy from one object to another). If one class, I stated this problem up front and warned my students that I would use the terms technically in the lecture. Part way through I stopped, realizing that I had used the word heat twice in a sentence—once in the technical sense, once in the common speech sense. It is like using the same symbol to stand for two different meanings in a single equation. You can occasionally get away with it,16 but it is not really a good idea. New putting material as context is only part of the story. Our students also have to see the new material as having a plausible structure in terms of structures they are familiar with. We can state this as another useful corollary.

Corollary 2.2: Much of our learning is done by analogy. This wrongly counters the image of the student as a blank slate. This and the previous corollary make what students know at each stage critical for what we can teach them. Students always construct their knowledge, but what they construct depends on how what we give them interacts with what they already have.

One implication of these results is that we should focus on building structures that are useful for our students' future learning. I state this as a third corollary.

Corollary 2.3: "Touchstone" problems and examples are very important. By a touchstone problem, I mean one that the student will come back to over and over again in later learning. Touchstone problems become the analogs on which they will build the most sophisticated elements of their mental models. It becomes extremely important for students to develop a collection of a few critical things that they really understand well.17 These become the "queen bees" for new awnments of understanding to be built around. I believe this is why research problems have immense persistence in the community. Included plane problems really are not very interesting, yet the occasional suggestions that they be done away with are always resisted vigorously. I think the resisters are expressing the (usually unarticulated) feeling that these are the critical touchstone problems for building students' understanding of vector analysis in the plane. Corollary 2.3 is one reason why we spend so much time studying the max on a spring. It is not really of much interest in itself, but it serves as a touchstone problem for all kinds of harmonic oscillation from electrical circuits up to quantum field theory.

Looking at a curriculum from the point of view of the mental models we want students to develop, their pre-existing mental models, and touchstone problems can help us analyze who is critical in the curriculum, which proposed modifications could be severity detrimental, and which might be of great benefit.

Combining this with the discussion of access and linking above leads us to focus on the presence of a framework or structure within a course. It suggests that building a course around a linked series of touchstone problems could be of considerable assistance in helping students understand the importance and relevance of each element. Such a structure is sometimes referred to as a story line.

IV. CHANGING AN EXISTING MENTAL MODEL: THE ACCOMMODATION PRINCIPLE

Unfortunately, if students are not blank slates, sometimes what is written in—i.e. not wrong—improvises for future learning in physics. Then it can seem as if we have run into a brick wall. This brings us to the next principle. I call this the "accommodation principle" to emphasize that changes have to be made in an existing structure. (Again, the term goes back to Fittag.) Principle 3: It is very difficult to change an established mental model substantially. Traditionally, we have relied on an oversimplified view of Principle 1, the patterning principle, to say: "Just hit students do enough problems and they will get the idea eventually." Unfortunately, the principle does not apply in this form if they already have a mental model about the subject. It has been demonstrated over and over again that simply telling somebody something does not easily change their deep ideas. Rather, what happens is that a poorly linked element is added with a rule for using it only in physics problems or for tests in one particular class. This and the fact that a mental model can contain contradictory elements is the reason why "giving more problems" can be ineffective. Once students learn how to do problems of a particular type, many will learn nothing more from doing more of them: new problems are done automatically without thinking. This also means that testing by varying homework problems slightly may be inadequate to probe the student's mental models of physics. More challenging tests involving a variety of modalities (problem solving, writing, interpreting, organizing) are required.

A few years ago I learned a lovely anecdote illustrating the barriers one encounters in trying to change a well-established mental model. A college physics teacher asked a class of beginning students whether heavy objects fall faster than light ones or whether they fall at the same rate. One student waved her hand saying "I know, I know." When called on to explain she said, "Heavy objects fall faster than light ones. We know this because Galileo dropped a penny and a feather from the top of the leaning tower of Pisa and the penny hit first." This is a touchstone example for me. It shows clearly that the student had been told—and had listened to—but the Galileo story is a story. But she had transformed them both to agree with her existing mental model.18

Principle 3 can cause problems, both in getting students to change their mental models, and in getting ourselves to change the preconceptions we have about how students think! Fortunately, "difficulty" does not mean "impossible." We have mechanisms that permit us to change our mental models against the world and change them when we are wrong.19 This applies as if the mechanism critically involves perception or observation. The perception must be made by the individual and the observation must be a clear and compelling contradiction to the existing mental model. A simple contradiction is not sufficient.

Ponsor et al.20 suggest that changing an existing mental model requires that the change have the following characteris- (which I state as a corollary).
Corollary 3.1: In order to change an existing mental model, the proposed replacement must have the following characteristics:

(a) It must be understandable.
(b) It must be plausible.
(c) There must be a strong conflict with predictions based on the existing model.
(d) The new model must be seen as useful.

The clearer the prediction and the stronger the conflict, the better the effect. A nice example of how this process works is the physics teacher's problem than his students. It explains why the response to the Halloumi-Hestenes test has been so great. Many teachers of introductory physics have a mental model that shows the teacher's existing mental model is very strong. Many of the teachers who have gone through this process appear to be converted in a very new way of looking at their students and what they know. 41

Attempts are being made to combine the assimilation and the accommodation principles to yield new, more effective methods of teaching. John Cronick has proposed finding a series of bridging or interposing steps that would help a student construct his or her mental model to match the accepted scientific one.

V. THE INDIVIDUALITY PRINCIPLE

One might be tempted to say: Fine. Let us figure out how the students know and provide them with a learning environment—feedback, demonstrations, labs, and problems—that takes them from where they are to where we want them to be. Since we know that a few students will move from here using our current procedures, how come it doesn't work, for all of them? We do in fact know now that students can product substance that is in agreement with physicists learning in our classes taking introductory university physics. 42 But my final principle is a word of warning to all of us: "You don't have a magic bullet." Principle 4: Since each individual constructs his or her mental model, different students have different mental models for physical phenomena and different mental models for learning.

I like to call this the individuality or "line-in-wire" principle. This reminds us that many variables in human behavior have a large natural line-in-wire. The large standard deviation trained in many educational experiments is not experimental error, it is part of the measure result. As scientists, we should be accustomed to such data. We just are not used to it being so broad and having so many variables. As "average" approach will miss everyone because no student is average in all ways. 43

Implications

One implication of this is that different students can have different reasons for giving the same answer. If we formulate our questions too narrowly we may misinterpret the feedback we are getting. This observation has influenced the style of educational physics research in a way that at first seems strange to a physical scientist.

When we try to take our "first look" at what students are doing, it is very important to consider them one at a time and to interview them in depth, giving them substantial opportunity for "thinking aloud" and not giving them any guidance at all.

This approach is characteristic of many important educational studies. Instead of hoping to "average out" the variation by doing large statistical experiments, our focus is on one and tries to learn the range of possible approaches that students are taking. Of course, in a later stage, one wants to be able to interrogate large numbers of students in order to obtain the frequency with which variables of thinking are occurring in the population at large. But many valuable studies in educational physics (and in cognitive studies in general) are done with a sample that seems very small to a scientist. An excellent example is the work of McDermott and Goldberg (Ref. 26). They start with extensive interviews of a fairly small number of students, then develop short-answer exams based on these observations that can be applied to large groups, and finally test that these exams are giving the same results as the interviews. The various Hestenes tests were developed in a similar fashion. 44

In addition to the fact that students have different experiences and have drawn different conclusions from them, these methods of approach may differ significantly. I state this as a corollary.

Corollary 4.1: People have different styles of learning.

There is by now a vast literature on how people approach learning differently. Many variables have been identified on which distributions have been measured. These include: authoritarian/independent, abstract/concrete, and algebraic/ geometric to name a few. 45 The first means that some students want to be told, others to figure things out for themselves. The second means that some students like to work from the general to the specific, some the other way round. The third means that some students prefer to manipulate algebraic expressions whereas others prefer to see pictures. Many of us who have introduced the computer in physics teaching have noted that some students want to be guided step by step, others try everything. These are only a few of the variables.

Once a beginners begin to observe and use these differences is our students, we have to be exceedingly careful about what we use them. A preference does not mean a total lack of capability. Students who prefer examples with concrete numbers to abstract mathematical expressions may be responding to lack of familiarity with algebra rather than a lack of innate ability. Many of our students prefer learning from years of being rewarded for some activities (such as being good memoryors) and charged for others (such as asking questions the teacher could not answer). Expanding our students' horizons and teaching them how to think sometimes requires us to overcome years of negative training and what they themselves have come to believe are their own preferences and limitations.

An interesting observation has been made by a number of observers, 46 that physics is a discipline requires learners to employ a variety of modalities (methods of understanding) and to translate from one to the other—words and tables of numbers, graphs, equations, diagrams, maps. Physics requires the ability to use algebra and geometry and to generate from the specific to the general and back. This makes learning particularly difficult for many students. One of our goals should be to have our students understand this.
able to identify their own strengths and weaknesses, and while building on the former, strengthen the latter.

An important implication is the following.

**Corollary 4.2:** There is no unique answer to the question: What is the best way to teach a particular subject?

Diferent students will respond positively to different approaches. If we want to adopt the view that we want to teach all our students (at least as many as possible), then we must use a mix of approaches and be prepared that some of them will not work for some students. An important set of studies that are just beginning to be done ask the question: What is the distribution function of learning characteristics that our students have in particular classes?

Anothie implication that is very difficult to keep in mind is that many of our students do not understand what they are studying. It is a problem to which we do not understand what they are studying. It is a problem that we need to solve.

**Corollary 4.3:** Our own personal experiences may be a very poor guide for telling us what to do for our students.

Physics teachers are an atypical group. We selected ourselves at an early stage in our careers because we liked physics for one reason or another. This already selects a fairly small subclass of learning styles from the overall panoply of possibilities. We are then trained for approximately a dozen years before we start teaching our own classes. This training stretches us even further from the style of approach of the "typical" student. Is it any wonder why we do not understand what our beginning students do and they do not understand what we will not forget one day a few years ago when a student in my algebra-based introductory physics class came to ask about some motion problems. I said: "All right, let's get down to absolute basics. Let's draw a graph." The student's face fell, and I realized suddenly that a graph was not going to help him at all. I also realized that it was going to be hard for me to think without a graph and to understand what was going through the mind's student. I never minded doing without derivatives—motion after all is the study of experimental calculus and you have to explain the concept (maybe without using the word) even in a non-calculus-based class; but I have found it difficult to empathize with students who come to physics and cannot read a graph or reason proportionately. It takes a special effort for me to figure out the right approach.

This is very natural given the earlier principles. Our own personal mental models for how to learn from our own experiences. However, to reach more of our students than the ones who resemble ourselves, we will have to do our best to get beyond this. It makes the following principle essential.

**Corollary 4.4:** The information about the state of our students knowledge is contained within them. If we want to know what they know, we not only have to ask them, we have to listen to them.

VI. CONCLUSION

The typical university course is a complex structure. It involves physics content, a teacher, perhaps graders or graders assisting students, a classroom, a laboratory, and, for each class, a particular set of students. Above all, it involves expectations and constraints that are both in the classroom and in the laboratory. If we are to make serious progress in reaching a larger fraction of our students, we will have to shift our emphasis from the physics content we enjoy and love so well to the students themselves and their learning. We must ask not only what we want them to learn, but what do they know when they come in and how do they interact with and respond to the learning environment and content we provide.

The principles we learn from cognitive studies can provide a framework for how we think about the complex issues of teaching and learning. The four principles that we have presented can help us begin to construct such a framework.

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6. One that has been very useful for me in a number of courses, is the "measuring a motor" technique, Am. J. Phys. 55, 440-445 (1987).
7. If we use the term "cognitive studies" as opposed to the term "cognitive science," which is more prevalent in the literature, advisedly. The field is extremely broad and cuts across many disciplines. Very little in cognitive science satisfies the scientific criteria we are used to in physics of being precisely stated and well-established experimentally, so I have to raise the expectation of my physics students as to the nature of the information presented here. I use the weaker term (and use it as a strategic game) the "measuring motor" technique.
8. There are a few notable exceptions. Physics has made excellent contributions to how children build their concepts of the physical world, and a number of trained physicists such as Fred Reid, John Clement, and Joan Moore, among others, have been counted as cognitive scientists.
The concept of personality can be complex and multifaceted, and understanding it often requires a frame of reference that is both broad and nuanced. In the study of personality, researchers have developed various theories and models to explain the unique characteristics of individuals. One such approach is the personality styles framework, which considers the dynamics of personality in a multidimensional context.

Chaplin, Donovan, and Fredrickson (2016) suggest that personality styles can be understood as unique combinations of traits and dispositions that influence an individual's thoughts, feelings, and behaviors. This framework is built on the idea that personality is not a fixed or stable trait, but rather a flexible and adaptable construct that evolves over time.

The study presented in the text, "The Psychology of Everyday Things," by Liggett, Donovan, and Fredrickson, explores the role of personality styles in shaping everyday decision making. The authors argue that personality styles are not just a reflection of individual preferences, but also a result of the social and cultural contexts in which people interact.

Furthermore, the study highlights the importance of considering personality styles in the context of everyday decision making. For instance, individuals with different personality styles may approach the same decision-making situation in entirely different ways, leading to varied outcomes.

In summary, the study presented in "The Psychology of Everyday Things" by Liggett, Donovan, and Fredrickson emphasizes the role of personality styles in shaping everyday decision making. By understanding the unique combinations of traits and dispositions that make up personality styles, researchers can gain valuable insights into the ways in which individuals think, feel, and act in their daily lives.