Engineering Introduction in Pre-Calculus Courses

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Introduction

The freshman engineering program at the University of Tennessee consists of two 4 hour courses, Physics for Engineers I and II. Calculus I is a co-requisite for Physics for Engineers I. The content of the Physics for Engineers courses is an introduction to engineering physics (approximately the first 20-25 chapters of an introductory engineering physics textbook), and an introduction to elements of successful engineering practice (teamwork, engineering design, and communication). Significant effort has been put into these courses, and the courses have been shown to increase retention in engineering (Parsons et al, 2002; Parsons et al, 2008).

Of the 565 freshman engineering students at the University of Tennessee in Fall 2011, 23% were placed in a pre-calculus course their first semester during orientation advising. Nine percent of the students placed in pre-calculus were successful in bumping themselves up to the first calculus course through either AP credit, taking a pre-calculus course during the summer, or passing a math placement test. This left 21% of the incoming freshman engineering class in a pre-calculus course. Since the first calculus course is a co-requisite for the first engineering course, the students in the pre-calculus course had no contact with the college of engineering in the fall semester. Only 64% of these students remained in engineering, taking calculus and the first engineering course in spring semester. Indeed, almost 45% of the students that leave the College of Engineering have never taken an engineering course. In order to improve retention of the pre-calculus students, three intervention techniques were used in Fall 2012: a learning community was offered to these students, students were placed together in pre-calculus sections, and an engineering professor visited the pre-calculus classes about every other week. The first two interventions were aimed at creating a community and encouraging cooperation among the students, a proven good practice in education (Chickering and Gamson, 1987). The third intervention was aimed at showing relevance of the mathematics to engineering, which has been successfully used in calculus courses (Neubert et al, 2011). The engineering professor would spend five to ten minutes showing an engineering application of the math concepts that were being covered in the class. Relevant announcements concerning college of engineering events and information on advising were also a part of the visit. The goal was to make students feel a part of the college of engineering.

This paper provides an overview of the third intervention, that of the engineering professor visiting the pre-calculus classes. The engineering applications that were covered in the pre-calculus class are given, along with preliminary results on the effectiveness.
Engineering Applications in Pre-Calculus

A total of seven visits were made to the pre-calculus classes by an engineering professor. The professor that visited the class was Dr. Richard Bennett, the Director of Engineering Fundamentals. By having the same professor visit each time, the students became comfortable with the professor. In each visit, an engineering application of the math concept that was currently being covered was presented. The following provides a brief description of each of the topics and presentations. Full details of each presentation are at http://ef engr.utk ed/riser/pre-calculus/math-130/.

Algebra and quadratic equations: Since the engineering instructor is a structural engineer, he began the visits by describing what he did in his professional practice. The engineering illustration was design of a reinforced concrete beam. The formula to determine the required area of the reinforcement steel is:

\[
\frac{w_ul^2}{8} = 0.9A_yf_y \left( \frac{d - 1}{2A_yf_y} \right)
\]

where \( w_u \) is the load = 200 lb/in; \( l \) is the length of the beam = 360 inches; \( f_y \) is a material property of the steel called the yield strength = 60000 pound/in\(^2\); \( d \) is the distance from the top of the beam to the reinforcement = 27 inches; \( f'_c \) is the compressive strength of the concrete = 4000 pound/in\(^2\); and \( b \) is the width of the beam = 16 inches. The students were asked if they could solve for \( A \), which is done using a quadratic equation. Of course there are two solutions, 2.34 in\(^2\) and 43.74 in\(^2\). The students were asked which one was correct, and most chose the largest value. This is the incorrect value, as the equation is only valid up to 12.31 in\(^2\). Admittedly, it was a trick question to ask which was the correct solution without giving full information, but it was pointed out to students that one of the important things to know is the range of applicability of equations, and math problems which the range of \( x \) is representative of real life problems.

Lines: A standard digital bathroom scale was brought into class, and students were asked if they knew how it worked. The instructor had partially disassembled the scale, so students could see the small cantilever beams that were used to support the scale, and the strain gages attached to the beams. The students were told how there was a linear relationship between load and strain, which was a function of the modulus of elasticity, or the slope of a line. The students were also told about how strain gages work, and the importance of a gage factor, which is also just the slope of a line. Students were told how linear relationships were the basis for many everyday objects.

Functions: The time to drain a tank was used as an example of functions, where the time to drain a tank is:

\[
h(t) = h_0 - \frac{A_1}{A_2} \sqrt{2gh_0} t + \frac{g}{2} \frac{A_1^2}{A_2} t^2
\]

where \( A_1 \) is the cross-sectional area of the tank, \( v_1 \) is how fast the level of the tank is dropping, \( A_2 \) is the area of the drain, \( v_2 \) is how fast the water is coming out of the drain, \( h \) is the height of water above the drain, \( h_0 \) is the initial height of the water, \( t \) is time, and \( g \) is the
acceleration due to gravity. The students were asked what does the terminology \( h(t) \) mean, as some students think this means \( h \) multiplied by \( t \). Students were asked if they could find the time to drain the tank for given parameters, and also if they could find the time to drain half the tank. The instructor looked at a plot of \( h(t) \) with the students to show what was physically happening, including looking at the slope of the graph, which showed how fast the water was draining.

**Rational functions:** Measuring speed using the Doppler effect was used to illustrate rational functions. The equation to determine the speed of an object based on the frequency shift is:

\[
v_s = \frac{(f' - f_0)}{2f_0 + (f' - f_0)}v
\]

where \( f_0 \) is the initial frequency, \( f' \) is the shifted frequency, \( v \) is the speed of sound (usually taken as 767 mph), and \( v_s \) is the speed of the object. A plot of the speed of the object vs. the frequency shift was shown to the class. Two aspects of the graph were discussed. Initially the graph is close to linear since the frequency shift is small, and the \( 2f_0 \) term dominates the behavior of the denominator. As the frequency shift increases, it starts to dominate the behavior of the denominator, and the speed of the object asymptotically approaches the speed of sound.

**Logarithmic functions:** Altimeters were used to illustrate logarithmic functions. The engineering application started with just talking about fluid pressure, and the pressure on Hoover Dam. The discussion then went into determining air pressure, and how air was a compressible fluid. The altitude is approximated by:

\[
z = 3.28\left(\frac{RT}{gM}\right)\cdot\log_e\left(\frac{p_o}{p}\right)
\]

where \( z \) is the height above sea level in feet, \( R \) is the gas constant, \( T \) is temperature of the air measured in Kelvin, \( g \) is the acceleration due to gravity, \( M \) is the molar mass of air, \( p_o \) is the atmospheric pressure at the sea level and \( p \) is the atmospheric pressure at the measurement height. For a temperature of 68°F, the equation becomes \( z = 28087\cdot\log_e\left(\frac{p_o}{p}\right) \). An absolute pressure sensor was brought into class, and the altitude was determined based on the measured pressure. The calculated altitude was compared to the altitude of Knoxville, TN as given at [http://www.knoxvilledemographics.com/](http://www.knoxvilledemographics.com/). Due to the hilly nature of the campus, elevations of the classrooms for the different sections were about 50 ft different. This elevation change was easily seen in the difference in absolute pressure readings, which was interesting to the students. Finally, a plot of \( z \) vs. \( p_o/p \) was shown and discussed.

**Trigonometric functions:** Trigonometric functions were illustrated using sound. Specific frequencies were played using the web site [http://www.falstad.com/fourier/index.html](http://www.falstad.com/fourier/index.html). Students could both see and hear a sine wave. Sine waves were then added to show what the resulting sound wave looked like, and also what it sounded like. Students were then conceptually introduced to the reverse process, or Fourier transforms, in which an arbitrary wave is broken down into sine and cosine components. To illustrate this, the iPad app SpectrumView was used. This app shows a continuous spectrum of sound waves. A tuning fork was held near the iPad microphone to show the frequencies of the tuning fork. The fundamental frequency was very
clear, but higher frequencies were also present, which lead to an interesting discussion of vibrations.

**Law of sines and cosines:** The final application was law of sines and cosines, and relative velocity was chosen for this application. The traditional airplane problem was used, with the specific example being an airplane can maintain an air speed of 380 mph and needs to fly to a city that is 500 miles @ 20° E of N of where it starts. The wind is blowing at 70 mph in a direction of 40° W of N. Determine the direction the airplane needs to head to reach the desired destination, and how long it will take to make the trip. A vector diagram was drawn, and the students were shown how the law of sines and cosines could be used to determine the required information.

**Preliminary Results**

In Fall 2012, there were 620 freshman engineering students at the University of Tennessee. Initially 35% (217 students) of these students were placed in a pre-calculus course. After adjustments of dual enrollment, AP credits or math placement results, 21% (133 students) of the freshman engineering class placed in a pre-calculus course. Four special sections of pre-calculus were established for these students. The sections were called RISER sections, named after the title of the grant, Research and Instructional Strategies for Engineering Retention. The 133 students were strongly encouraged to enroll in one of four RISER sections of pre-calculus. 94 students, or 71% of the eligible students, did enroll in a RISER section. Some students were unable to enroll in RISER sections due to conflicts with ROTC and band. A small number of students simply chose not to enroll in RISER sections. One student did not register for a math class.

Two math instructors were chosen to teach the RISER sections, each teaching two sections. The two instructors were specifically chosen based on their proven teaching ability. There were several meetings between the math instructors and the college of engineering prior to the beginning of the semester to work out schedules and visits to the classes. Both instructors were very supportive of visits by an engineering professor.

There were a total of 18 sections of the pre-calculus course taught in the 2012 fall semester, with four of these being RISER sections reserved for freshman engineering students. A common final exam was given to all sections. The overall average on the final exam was 74.8. The average on the final exam of the RISER pre-calculus sections for engineering students was 80.7. It is difficult to isolate the reason for the higher average, but we believe it is a combination of better students, excellent instruction, and increased student engagement.

Table 1 summarizes the results from the Fall 2012 pre-calculus course, which compares students in the RISER and non-RISER sections. The comparisons are given both in terms of actual numbers of students, and the percentages of the students in that group. The percentage of pre-calculus students moving on to an engineering course was about the same as the Fall 2011 for the non-RISER sections, but was significantly higher for the RISER sections. The pass rate was also
much higher for the RISER sections, resulting in increased overall retention, even if the students decided to leave engineering.

**Table 1. Summary of pre-calculus sections**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>RISER sections</th>
<th>Non-RISER sections</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of students</td>
<td>94</td>
<td>38</td>
</tr>
<tr>
<td>Enrolled in engineering course in spring semester</td>
<td>70 (74%)</td>
<td>25 (66%)</td>
</tr>
<tr>
<td>Did not pass pre-calculus</td>
<td>5 (5%)</td>
<td>8 (21%)</td>
</tr>
</tbody>
</table>

Table 2 summarizes the results of survey questions related to the impact of the classroom visits by Dr. Bennett. Overall, the students indicated there were positive aspects to the classroom visits.

**Table 2. Summary of results of student survey**

<table>
<thead>
<tr>
<th>Impact of classroom visits</th>
<th>Very negative</th>
<th>Negative</th>
<th>Neutral</th>
<th>Positive</th>
<th>Very Positive</th>
<th>Rating Average</th>
</tr>
</thead>
<tbody>
<tr>
<td>Enjoyment of course</td>
<td>0</td>
<td>2</td>
<td>11</td>
<td>23</td>
<td>8</td>
<td>3.84</td>
</tr>
<tr>
<td>Understanding of how your coursework relates to advances in science and technology</td>
<td>0</td>
<td>1</td>
<td>14</td>
<td>22</td>
<td>8</td>
<td>3.82</td>
</tr>
<tr>
<td>Your ability to provide examples of how engineers use research to solve &quot;real world&quot; problems</td>
<td>0</td>
<td>0</td>
<td>11</td>
<td>20</td>
<td>13</td>
<td>4.05</td>
</tr>
<tr>
<td>Your ability to see yourself as an engineer</td>
<td>0</td>
<td>2</td>
<td>18</td>
<td>14</td>
<td>11</td>
<td>3.76</td>
</tr>
<tr>
<td>Your passion for engineering</td>
<td>2</td>
<td>3</td>
<td>19</td>
<td>11</td>
<td>10</td>
<td>3.53</td>
</tr>
<tr>
<td>Your perception that your field of engineering is intellectually challenging</td>
<td>1</td>
<td>1</td>
<td>10</td>
<td>20</td>
<td>12</td>
<td>3.93</td>
</tr>
<tr>
<td>Your perception that your field of engineering is personally meaningful</td>
<td>2</td>
<td>3</td>
<td>14</td>
<td>15</td>
<td>10</td>
<td>3.64</td>
</tr>
<tr>
<td>Your perception that your field of engineering is beneficial to society</td>
<td>2</td>
<td>0</td>
<td>11</td>
<td>15</td>
<td>16</td>
<td>3.98</td>
</tr>
<tr>
<td>Your sense of connection or partnership with at least one faculty member</td>
<td>0</td>
<td>1</td>
<td>24</td>
<td>17</td>
<td>1</td>
<td>3.42</td>
</tr>
<tr>
<td>Your determination to earn an engineering degree</td>
<td>0</td>
<td>3</td>
<td>18</td>
<td>12</td>
<td>10</td>
<td>3.67</td>
</tr>
<tr>
<td>Your desire to continue pursuing an engineering degree</td>
<td>1</td>
<td>5</td>
<td>14</td>
<td>13</td>
<td>10</td>
<td>3.60</td>
</tr>
</tbody>
</table>
**Lessons Learned and Continued Work**

The students seemed to enjoy having the visits from an engineering professor. Feedback is being obtained from the students, and is being analyzed by an outside assessment consultant. These students will continue to be monitored to see how well they do in the spring semester in Calculus and Physics for Engineers I. Some of the potential changes that are being considered for next Fall are as follows:

- We are considering pre-assigning students to the pre-calculus sections during orientation next summer. There were conflicts with ROTC and band that prevented some students from being in the special pre-calculus sections. Although overall the placement of students in these sections worked well, some additional coordination and tweaking will help the process.

- One of the math instructors awarded extra credit points to students who turned in a solution to the quadratic equation problem (design of reinforced concrete beam) and the law of sines and cosines problem (airplane relative velocity). A possibility is to have a small problem related to each application, and students receiving extra credit for working that problem, or even have it as part of their homework assignment. This will be discussed with the math professors.

- The math instructors indicated that a few of the students did not need pre-calculus, and would have been fine going directly into calculus. We are in the process of examining the profile of those students and seeing whether they can be identified during orientation. As part of the overall retention program funded by this NSF grant, there is also a 1.5 week summer math camp for those students who were close to qualifying for calculus (students who had a math ACT of 27, with a 28 being required for calculus). Perhaps some of the pre-calculus students should have taken advantage of the math camp to pass the math placement test and qualify for calculus in the fall semester.

**Conclusions**

Significant efforts have and are being expended in order to retain and help freshman engineering students to succeed once they were in the first engineering courses at the University of Tennessee. However, no efforts were being expended towards those students who did not qualify for the first engineering course in the beginning Fall semester, and we were using none of the proven practices for retention and engagement with the pre-calculus students. As a result many of the students left engineering before taking the first engineering course. The goal of the project was to use some of the same techniques that had proven effective in engineering classes in the pre-calculus sections.

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References